Basic Concepts on Game Theory and Applications in Service Systems

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Decision Analysis

- Decision: the act of making a choice between alternatives
- Alternatives
- States of nature or events: unknown factors that affect the outcome of the decision
  - Not under the control of the decision maker
  - Mutually exclusive, collectively exhaustive
- Possible outcomes of a decision: the combined effect (payoff) of a chosen alternative and the state of nature that alternative obtains
- Decision matrix: assigns an outcome for each pair (alternative, state of nature)

Source: Decision making using game theory, Anthony Kelly

Example

Problem: Consider a company that is looking for a Customer-relationship Management (CRM) cloud-based solution that best fits its needs taking into consideration requirements from both business and IT.

Step 1: List possible alternatives

- CSP -1 (CSP SaaS solution-public cloud)
- Hybrid-2 (CSP SaaS solution + custom, in-house product configurator- public cloud + traditional IT)
- Priv-3 (Software package, private cloud)

Step 2: Identify states of nature

- Large increase in demand
- Moderate increase in demand
- Slight increase in demand

Source: Designing your cloud decision framework, François Habryn, Bob Freese, IBM

Step 3: List the possible outcomes (decision matrix)

Alternatives	Demand		
	Large increase	Moderate	slight
CSP-1	6.000.000€	4.000.000€	-2.600.000€
Hybrid-2	2.500.000€	5.000.000€	-1.000.000€
Priv-3	2.000.000€	1.500.000€	1.200.000€

- **HOW do we represent uncertainty?**
- **HOW do we make the choice?**

In order to use a decision matrix to analyze a decision problem we need:

Information about how outcomes are valued

- (Criteria): rank the different alternatives

Information about which of the states of nature will be realized

- Probability distributions

The role of Utility in Decision Theory

- A **utility function** describes an agent’s preferences on alternatives
  - It is a way of assigning a number to every possible alternative such that more preferred alternatives get assigned larger numbers than less-preferred alternatives
  - Monotonically increasing
  - The agent’s objective is to maximize its utility function given a set of budget constraints
- A **utility function** *also* describes an agent’s preferences on wealth
  - Agents care about the utility the money provides instead of monetary values
  - Monotonically increasing (more wealth is preferred to less wealth)
  - The agent’s objective is to derive the optimal choice such that his **expected utility** is maximized
    - Choice under uncertainty

Example (utility for commuters)

- Consider a model where the commuter decides to drive or take the bus depending on whether he prefers one bundle of characteristics to the other
- Let the bundle (x_1, x_2) represent the values of the characteristics of driving (e.g. x_1: travel time, x_2: waiting time)
- Let the bundle (y_1, y_2) represent the values of the characteristics of taking the bus
- Consider a utility function of the form $u(x_1, x_2) = \beta_1 x_1 + \beta_2 x_2$, where β_1, β_2 are unknown parameters estimated by statistical techniques
- Each commuter seeks to choose between the two bundles subject to its constraints in order to choose the mode of transport

Source: Urban Travel Demand: A Behavioral Analysis, T. Domencich and D. McFadden

Expected Utility

- Used to analyze <u>choice under uncertainty</u>
  - Consider an agent playing a lottery with outcomes (wealth) $\boldsymbol{w_1, w_2}$, …
  - Lottery's outcomes are determined according to a probability distribution $\boldsymbol{p_1, p_2}$, …
  - The agent has a utility function $\boldsymbol{u(w)}$ for each level of wealth
- Expected value of a lottery

$$E(v) = p_1 w_1 + p_2 w_2 + \ldots$$

- Expected utility of a lottery

$$E(u) = p_1 u(w_1) + p_2 u(w_2) + \ldots$$

Tolerance for Risk

3 types of agents

- Risk averse
  - prefer the sure thing to lotteries with the same expected value
  - strictly concave utility function
- Risk neutral
  - rank lotteries according to their expected value
  - linear utility function
- Risk taking
  - prefer the lottery to a sure thing with the same expected value
  - Strictly convex utility function

Example

- Consider an agent with utility ***u(w) = log(w)*** that has to choose among two lotteries:
  - A: Get wealth $w_1=1250$ for sure
  - B: Get $w_1=1500$ with probability $p_1=0.75$ *and* $w_2=500$ with probability $p_2=0.25$
- *Question: which lottery does the agent prefer?*
- The **expected value** of each lottery is: *1250*
  - *(indifferent between the two lotteries)*
- The **expected utility** of each lottery is:
  - $E_A(u) = u(1250)=3.096$
  - $E_B(u) = 0.75u(1500)+0.25u(500)=3.056$
  - *Prefers sure gambling (risk averse – concave utility function)*

Decision Making Models

Decision-making under certainty **(GAMES OF SKILL)**

- One-player games
- The player has complete control over all outcomes
- Criteria for ranking alternatives (objective function)
    - Utility
    - Profit
    - Cost
- Goal: optimize utility within a set of constraints
- **Tool**: linear programming/optimization theory

Decision making under uncertainty

- One-player against nature **(GAMES OF CHANCE)**
- The player faces a probability distribution over the outcomes that can occur
- Goal: optimize expected utility
- **Tool**: optimization theory

GAMES OF STRATEGY

- Involve two or more players
- Each player has partial control over the outcomes
- Goal: optimize expected utility
- **Tool**: game theory

Game Theory

Game theory is a mathematical method for analyzing strategic interactions among players

- *Interdependence*: each player's actions will affect other players whose reactions will in turn affect their rivals

Non cooperative game theory: a game is a model of all the moves available to the players

Cooperative game theory

- Focuses on what groups of players, rather than individuals, can achieve
- Describes only the outcomes that result when players come together in different combinations (coalitions)
- A game is cooperative if the players can make binding agreements about the distribution of the payoffs
- Key criteria for the evaluation of the outcomes
  - Stability: no coalition of players should want to deviate from the solution
  - Fairness: Players should be rewarded for what they contribute to the coalition

Elements of a Non-Cooperative Game

1. Players: economic agents who make decisions
   - Rationality: players choose strategies that maximize their pay-off
2. Strategies: plan of actions available to a player
3. Payoffs: a player's gain (or loss) from particular strategies
4. Moving sequence
   - Simultaneous: players take actions at the same time
   - Sequential: each player takes action in a particular sequence
5. Information: knowledge each player has about the game
   - Complete (incomplete): each player's payoff function known to all
   - Perfect (imperfect): at each move the player with the move knows the full history of the play thus far
   - Symmetric (asymmetric): players have the same information regarding each other's moves/payoffs

Prisoners' Dilemma

- Game:
  - Two prisoners are held for questioning concerning a crime
  - Each prisoner must decide individually which strategy to choose (one-shot game)
- Strategies: Deny, Confess
- Pay-offs: years of imprisonment
- Pay-off matrix
- **Question**: Which strategy will each player choose?
  - 'Confess' is the dominant strategy for A and B

	B		
		Confess	Deny
A	Confess	3,3	1,4
	Deny	4,1	2,2

- *Paradox*: individual decision-making leads to an inferior outcome for the players in comparison with joint decision-making

Nash Equilibrium

- *Nash equilibrium* is an outcome of a game in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only his own strategy unilaterally
  - In a game of two players A and B, the pair of strategies $(s^*, g^*)$is a Nash equilibrium if s^* is optimal for A given g^* <u>and</u> g^* is optimal for B given s^*
- A game may have more than one Nash equilibria
- There are games that have no Nash equilibria (in pure strategies)

Examples

1. Nash equilibria: (Top, Left), (Bottom, Right)

		B	
		Left	Right
A	Top	2,1	0, 0
	Bottom	0, 0	1, 2

2. There are no Nash equilibria (in pure strategies)

		B	
		Left	Right
A	Top	0, 0	0, -1
	Bottom	1, 0	-1, 3

Mixed Strategies

- Pure strategy: a specific action that a player will follow in every possible situation in a game
- Mixed strategy: a player randomizes his strategies and assigns a probability to each choice
- If we allow mixed strategies then every game with a finite number of players in which each player can choose from finitely many pure strategies has at least one Nash equilibrium.

Example

- A's strategies: Top with probability p, Bottom with probability $1 - p$ $(p \neq 0,1)$
- B's strategies: Left with probability q, Right with probability $1 - q$ $(q \neq 0,1)$

		B	
		Left	Right
A	Top	0, 0	0, -1
	Bottom	1, 0	-1, 3

A's expected pay-off = $q\big(0p + 1(1-p)\big) + (1-q)\big(0p - 1(1-p)\big)$

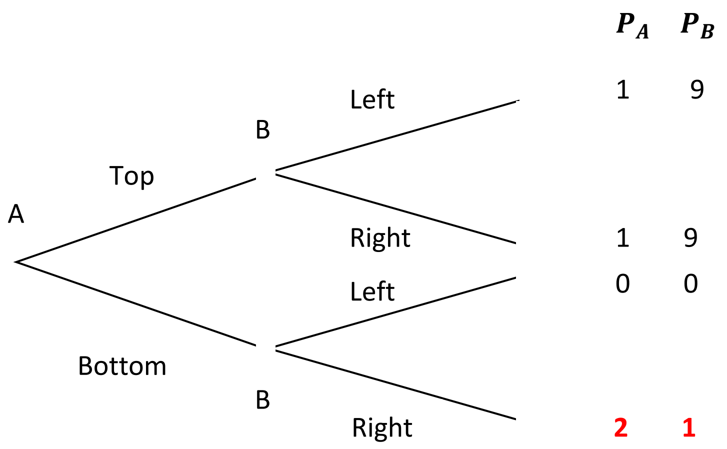
B's expected pay-off = $p\big(0q - 1(1-q)\big) + (1-p)\big(0q + 3(1-q)\big)$

- There are no Nash equilibria in pure strategies
- $(p, q) = (\frac{3}{4}, \frac{1}{2})$ is a Nash equilibrium in mixed strategies

Sequential Games

- Simultaneous game: Nash equilibria → (Top, Left), (Bottom, Right)
- <u>Sequential game</u>: A chooses first, B observes A and chooses next
    - (Top, Left) is not reasonable in the sequential game
- Extensive form of the game

	B	
A	Left	Right
Top	1, 9	1, 9
Bottom	0, 0	2, 1



Cooperative Games

Definition. Let $N = \{1, \dots, n\}$ be a finite set of players. A *coalition* S is defined to be a subset of N, $S \subset N$. The set of all coalitions is denoted by 2^N.

Notes.

1. $\emptyset$ is the empty coalition and N is the *grand coalition*.
2. The number of the elements of set 2^N is $|2^N| = 2^n$.
3. The set $N \backslash K = \{i \epsilon N, i \notin K\}$ is called *$K's$ complement*.

Definition. A *cooperative game* $G =< N, u >$ consists of two elements:

i. A set of players $N = \{1, \dots, n\}$
ii. A characteristic function u: $2^N \to \mathbb{R}$ specifying the value created by different coalitions of the players in the game and satisfying $u(\emptyset) = 0$.

Fundamental Research Questions in Cooperative Game Theory

1. What is the winning coalition?
2. How is overall value (collective gain) divided up among the various players?

Concepts and Models in Service Computing

Utility of an entity

- Utility models
  - *Individual decision making*
    - derive solutions so that utility is maximized
- Game theoretic models
  - *Strategic interactions among entities*
    - derive equilibria so that utility is maximized

Collective utility of a coalition

- Game theoretic models
  - *Collaborations among entities* derive coalitions that create the maximum collective utility
  - *Filter collective decisions* derive feasible solutions so that collective utility is improved

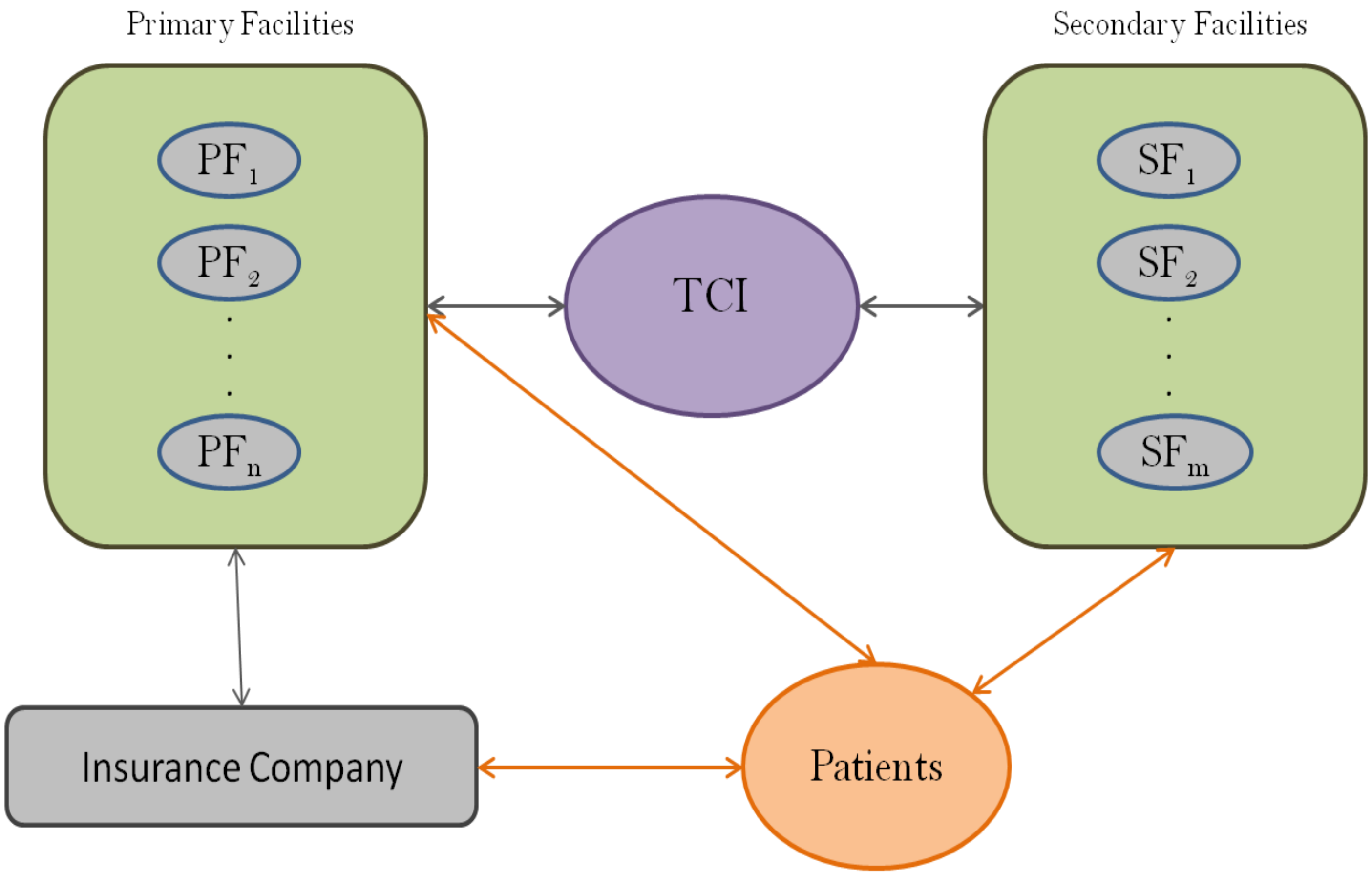
Application 1: Individual Decision Making in Healthcare

Medical Problem: Transition of care

- Manage patients transitions between healthcare settings in a cost-effective way and at the same time improve their medical status
- The transferring party provides medical records and instructions to the receiving party
- The receiving party monitors predefined patient characteristics and evaluates the health status of the patient
- The communication and the coordination of services between the two health care providers are performed by an intermediary
  - provides electronic health record technologies to gather, share and exchange information.

Source: A Service-oriented Approach for Improving Quality of Health Care Transitions, M. Bitsaki, Y. Viniotis

Service System



Key Performance Indicators

- Cost of care per patient
- Patient satisfaction
- Readmission rate from SF back to PF that reflects the quality of service provided by SF
- Market share each intermediary gains in a competitive environment

Factors that affect KPIs

- Services provided by the various entities
  - High quality services reduce readmission rate
- Use of Infrastructure (EMR of the patient, sensors in the patient room, automation in transition process)
  - Decreases readmission rate → decreases costs (indirectly)
  - Increases cost (directly)
- Matching of PFs to SFs
  - Considering patient preferences increases patient satisfaction
  - Considering geographical constraints reduces costs

The Problem of Matching Pfs to SFs

What is the **optimal matching** of PFs and SFs such that the overall cost is minimized provided that patients' preferences and economic constraints are satisfied?

Other research questions

1. What are the optimal strategies of the entities within the service system such that the performance of the system is improved in terms of customer satisfaction or readmission rate?

$\Rightarrow$ non-cooperative game

2. What are the optimal coalitions among SFs so that administrative costs are decreased under geographical and patient readmission constraints.

$\Rightarrow$ cooperative game

Application 2: Cooperative Games in Transportation

Problem

We consider a bus company that provides two routes for transporting passengers from pick-up point A to B and B to C respectively

- Consider that the Bus Manager has received a number of requests for trip A to B, a number of requests for trip B to C and a number of requests for trip A to C (path passengers)
- **Decision** to be made: which passengers to serve so that the collective utility is maximized

Source: Collective Utility in Hierarchical Structures of Collective Adaptive Systems: an Application in Transportation Systems, M. Bitsaki, M. Dramitinos, G. Koutras, D. Plexousakis, A. Psycharaki

Collective Utility

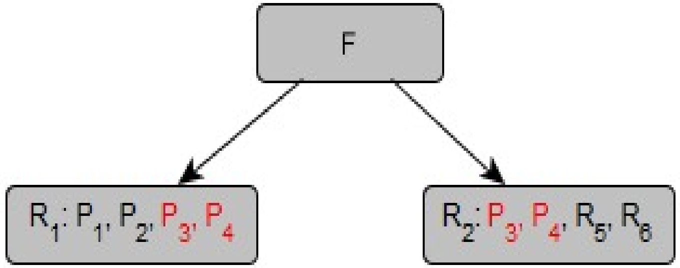
The **collective utility** of a coalition is a measure of the welfare of <u>all</u> players that participate in the coalition

$$v_E(a, w_1, \dots, w_k) = f_1(w_1, \dots, w_k)u_1(a, w_1) + \cdots + f_k(w_1, \dots, w_k)u_k(a, w_k)$$

a: utility parameters
w_i: player i preferences
u_i: individual utility for player i
f_i: weight for player i

Why Collective Utility is Important

- Evaluate coalitions
- Make collective decisions
- Improve the performance of service systems



$$u_i(t) = e^{\left(-\frac{k_i t}{w_i}\right)}$$

$$v_1 = \frac{w_1 u_1 + w_2 u_2 + w_{31} u_3 + w_{41} u_4}{w_1 + w_2 + w_{31} + w_{41}}$$

$$v_2 = \frac{w_5 u_5 + w_6 u_6 + w_{32} u_3 + w_{42} u_4}{w_5 + w_6 + w_{32} + w_{42}}$$

w_i: maximum travel time of passenger *i*

k_i: risk tolerance

u_i: utility

S_j: set of passengers in route R_j

$S_1' = S_1 - (S_1 \cap S_2)$ and $S_2' = S_2 - (S_1 \cap S_2)$

$$v = \frac{|S_1'|}{|S_1' \cup S_2'|} v_1 + \frac{|S_2'|}{|S_1' \cup S_2'|} v_2$$

Decision to be made: which passengers to serve so that the collective utility is maximized

Optimization Problem

$$\max_{K \subseteq N} u_K$$

$$s.t. |N_1 \cup N_3| \leq C_1$$

$$|N_2 \cup N_3| \leq C_2$$

$$w_{i1} + w_{i2} \leq w_i \; for \; each \; i \in \; N_3$$

$$T_1 \leq w_i \; for \; each \; i \in N_1$$

$$T_2 \leq w_i \; for \; each \; i \in N_2$$

$$T_1 + T_2 \leq w_i \; for \; each \; i \in N_3$$

u_K: collective utility of set of passengers K

$N = N_1 \cup N_2 \cup N_3$ set of all passengers

$C_1, \; C_2$ bus capacities

w_i: maximum travel time

$T_i, \; i = 1,2$ expected travel time

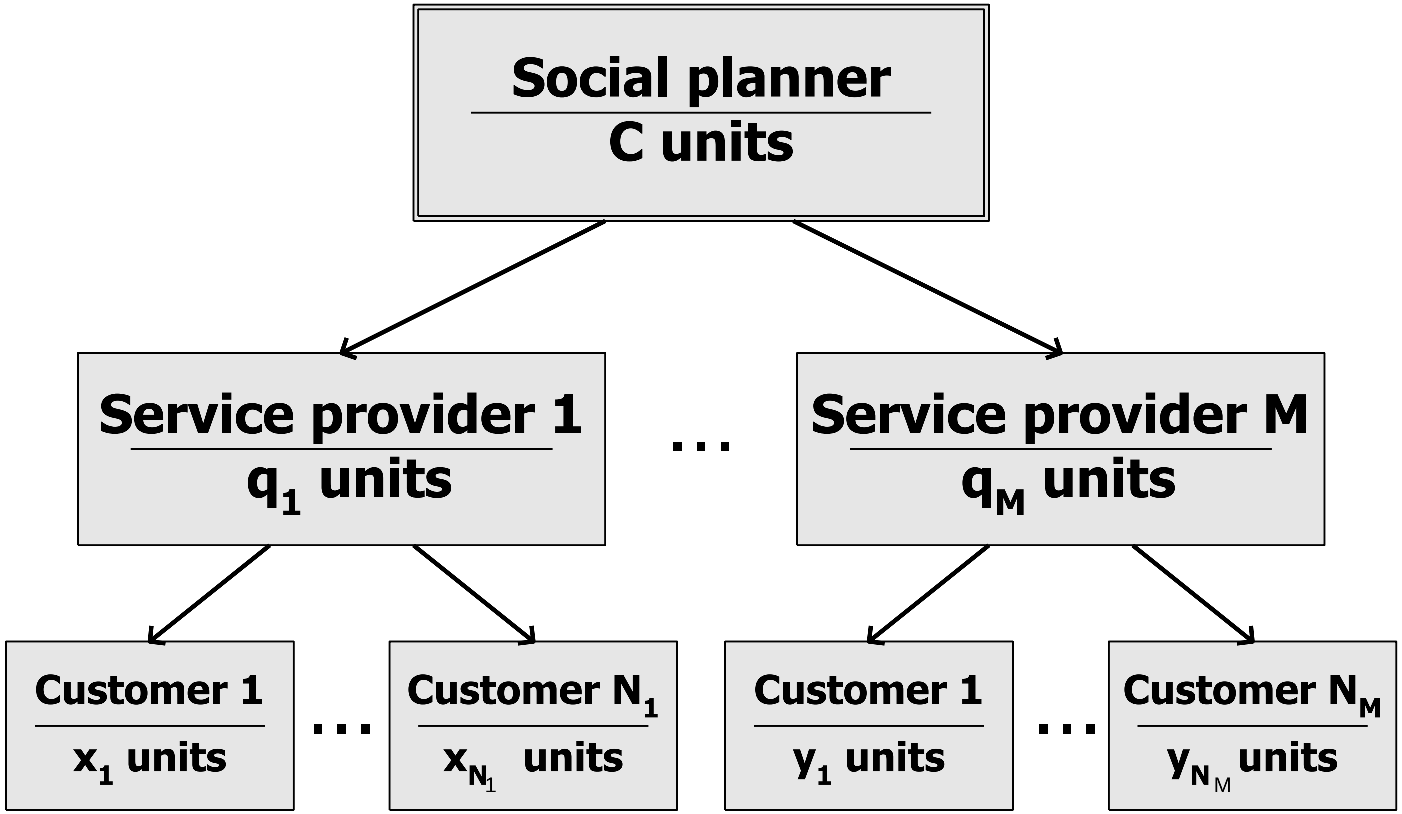
Solution: Hierarchical Approach

- We consider that the Bus manager has access only to path requests which then forwards to the lower level route managers with the additional information of how to split the preference w_i for each such request
- Each route manager has access to the information related to the requests made for his own route
- Each route manager solves the above mathematical problem considering only passengers of his own route
  - some passengers that want both routes may be accepted by route 1 but may not be accepted by route 2
  - path passengers that have won in both links are accepted. The available seats are offered to single route passengers provided that overall collective utility is maximized

Application 3: Auction Mechanisms in Resource Allocation

Problem

Sell C units of bandwidth in a single communication link to a set of N customers through M service providers



Assumptions

- Capacity C known to all players
- Each provider's population of local customers is fixed
- Each customer's marginal valuations for bandwidth units are diminishing and privately known to him
  - Example: $u_{1,1} = 10$, $u_{1,2} = 7$, $u_{1,3} = 3$
- Each provider's marginal valuations for bandwidth units equal the potential revenues from selling them locally, which are <u>not</u> known before the trade
- Social planner's objective:
  - efficiency $\rightarrow$ maximization of social welfare

THANK YOU!