Black Holes, Wormholes & Entanglement

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Agenda

Entanglement

Einstein’s Field Equation

Black Holes

Properties of Black Holes

ER = EPR

Summary

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- Entanglement
- Einstein's Field Equation
- Black Holes
- Properties of Black Holes
- ER = EPR
- Summary

Definition

$|\varphi\rangle \in \mathbb{H}_1 \otimes \cdots \otimes \mathbb{H}_n$ is *separable* $:\Leftrightarrow$

$|\varphi\rangle = |\psi_1\rangle \otimes \cdots \otimes |\psi_n\rangle$ with $|\psi_i\rangle \in \mathbb{H}_i$ for $1 \leq i \leq n$

$|\varphi\rangle$ is *entangled* $:\Leftrightarrow$ $|\varphi\rangle$ is not separable

Examples

$$\left|\varphi^{+}\right\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle)$$

$$\left|\varphi^{-}\right\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle - |1\rangle \otimes |1\rangle)$$

$$\left|\psi^{+}\right\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle + |1\rangle \otimes |0\rangle)$$

$$\left|\psi^{-}\right\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle - |1\rangle \otimes |0\rangle)$$

$$\left|W_{n}\right\rangle = \frac{1}{\sqrt{n}}\Big(\left|0...001\right\rangle + \left|0...010\right\rangle + \left|0...100\right\rangle + ... + \left|1...000\right\rangle\Big)$$

$$|\Psi^{n}_{GHZ}\rangle = \frac{1}{\sqrt{2}}\left(|0\cdots0\rangle + |1\cdots1\rangle\right)$$

A Phenomenon

$$\frac{1}{\sqrt{2}}\left(|00\rangle + |01\rangle\right) = |0\rangle \otimes \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

(separabel)

$$\frac{1}{\sqrt{2}}\left(\left|00\right\rangle + \left|01\right\rangle\right)$$

Measuring the first qbit results in $\left|0\right\rangle$ with probability 1. The second qbit will be measured as $\left|0\right\rangle$ or $\left|1\right\rangle$ with probability 1/2

This state is *entangled*

$$\frac{1}{\sqrt{2}}\left(\left|00\right\rangle + \left|11\right\rangle\right)$$

Measuring the first qbit results in $\left|0\right\rangle$ or $\left|1\right\rangle$ with equal probability. After that the value of the second qbit is already determined!

EPR Paradoxon

$$\frac{1}{\sqrt{2}}\left(\left|00\right\rangle+\left|11\right\rangle\right)$$

Measuring the first qubit results in $|0\rangle$ or $|1\rangle$ with equal probability. After that the value of the second qubit is already determined!

Einstein–Podolsky–Rosen Paradoxon
(1935)

…independent of the physical distance at the time of measurement!

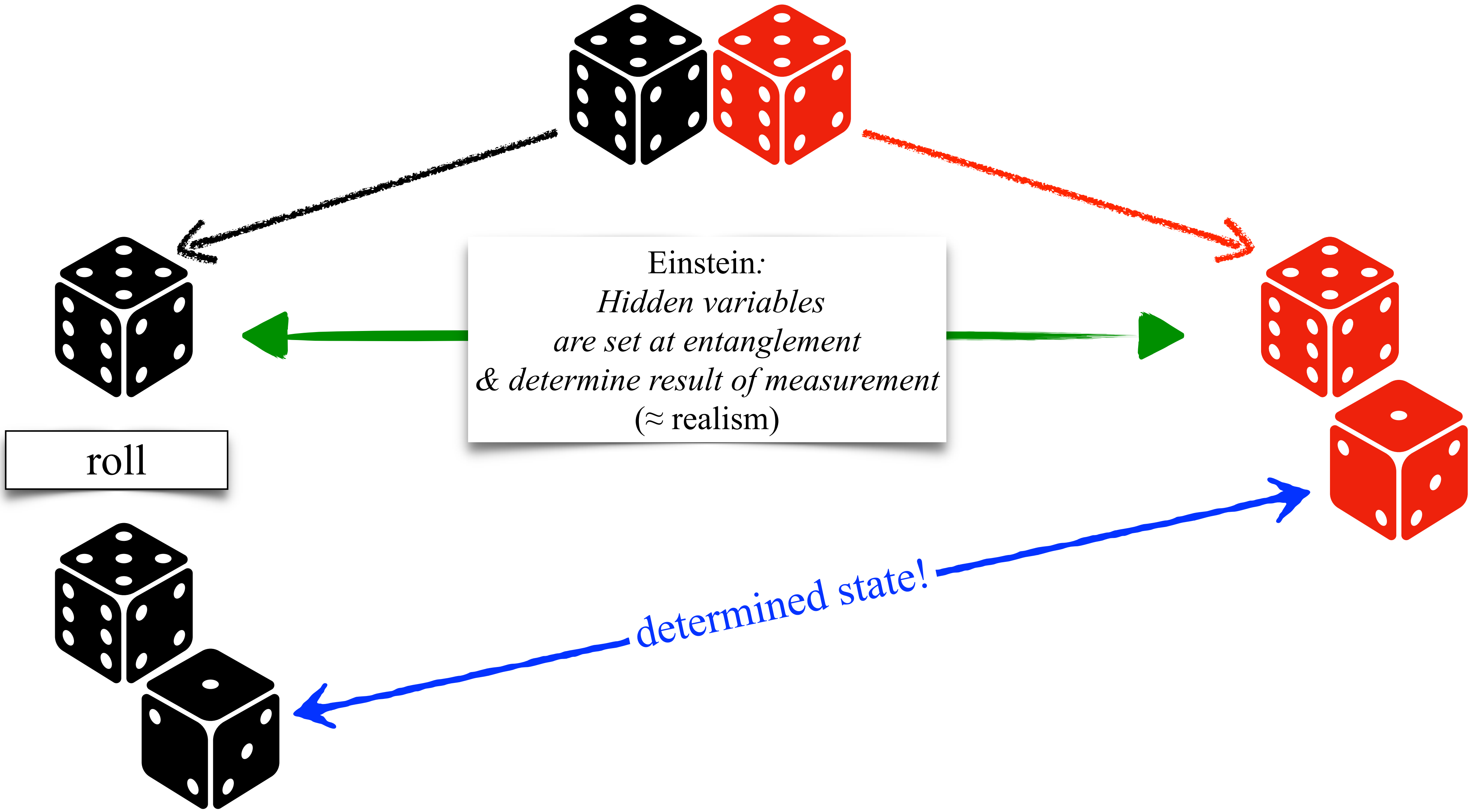
⇒ No communication, no interaction can take place between the qubits (speed of light limits the distance at which communication and interaction can take place)!

This is called *non-local*

Physics known by then <u>was</u> local!

Einstein:
Spooky actions at a distance

Intuition

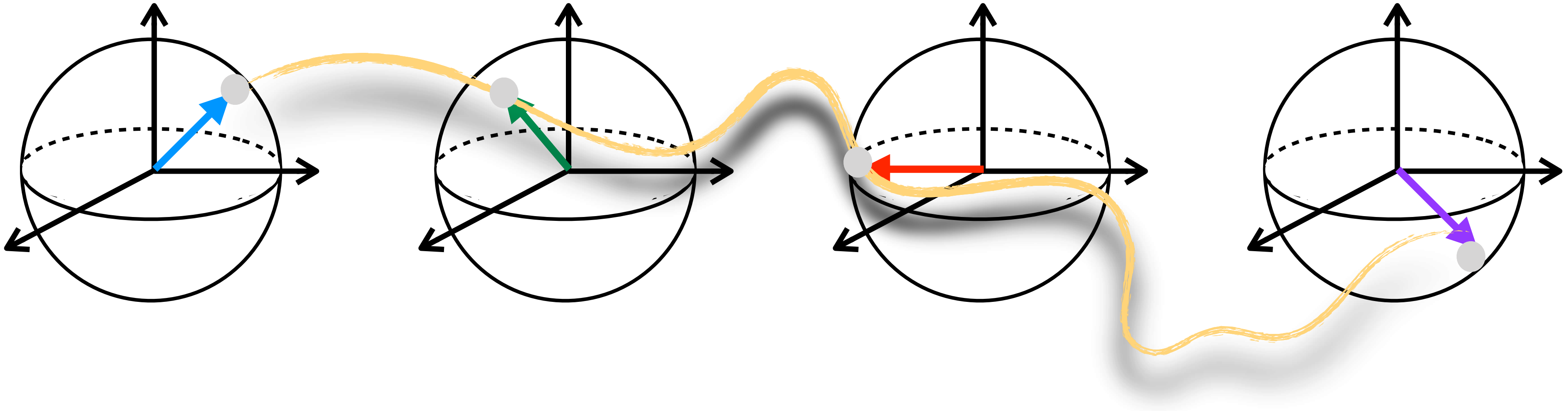


Einstein:
Hidden variables
are set at entanglement
& determine result of measurement
(≈ realism)

roll

determined state!

Entanglement as Global Phenomenon



Entanglement: Importance

Entanglement is unique for quantum computing!

Each computation not involving entangled qubits,
can be realized classically and **in principle** with the same
efficiency than a quantum computation

(**but**: n qubits ⇒ 2^n classical storage | quantum parallelism | …)

Every quantum algorithm showing exponential speedup
compared to a classical algorithm, must exploit entanglement.

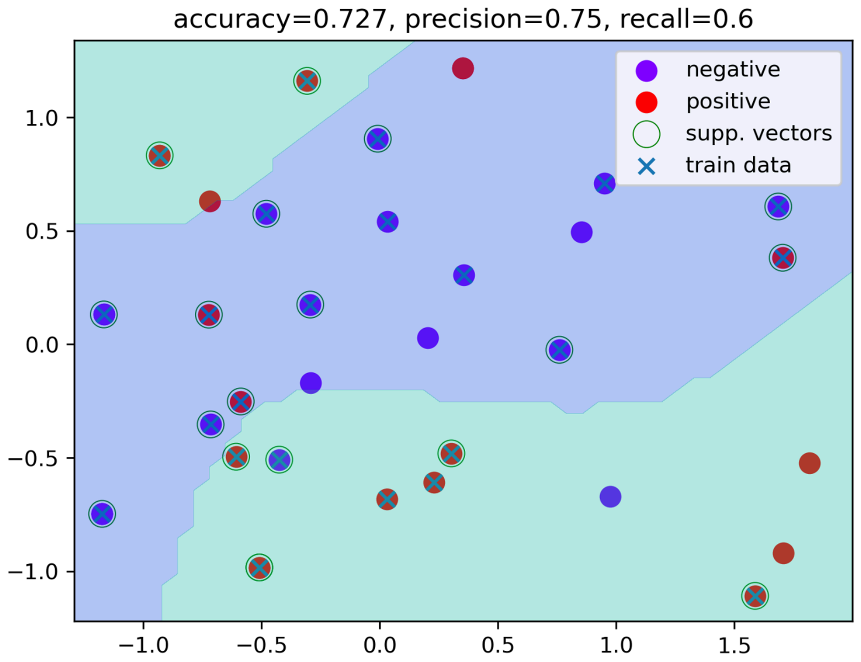
(R. Jozsa, N. Linden: On the role of entanglement in quantum computational speed-up.
(2003) arXiv:quant-ph/0201143v2)

Not Only Speedup

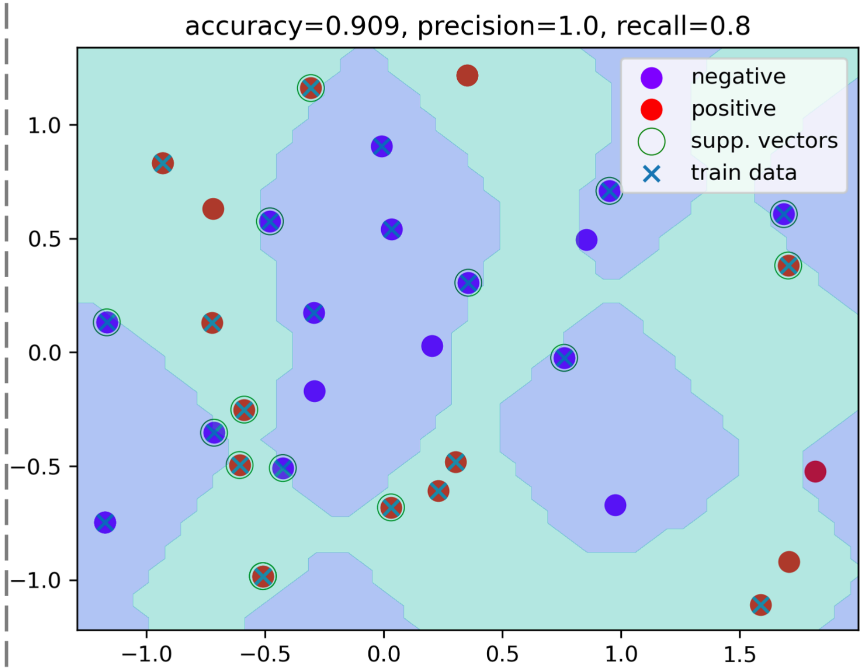
Most often, speedup is highlighted as quantum advantage

But precision can also be enhanced

- E.g. in classification



Classification result with classical SVM
(Radial Basis Function (RBF) Kernel)



Classification result with quantum SVM
(Quantum Kernel Estimation (QKE))

…And More…

The more test data are used, the smaller is the average error in classical supervised learning

The average error in quantum supervised learning is

$$Risk \geq 1 - \frac{r^2 n^2 + d + 1}{d(d+1)}$$

r: Schmidt-rank of training data n: Cardinality of S (training data) d: Dimension of Hilbert space

r = d : training data is maximal entangled

Already n = 1 implies : $\text{Risk} \geq 1 - \dfrac{d^2 + d + 1}{d(d+1)} \xrightarrow{d \to \infty} 0$

A single maximal entangled element of training data suffice in high dimensions to learn with low risk a unitary transformation

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Frequency of Entanglement

Let $\mathscr{H}$ be a Hilbert space with $\dim \mathscr{H} = d = 2^N$ and let $\mathfrak{D}, \mathfrak{S} \subseteq \mathscr{H}$ be all states or all separable states (mixed states)

Then, $\dfrac{\text{vol } \mathfrak{S}}{\text{vol } \mathfrak{D}}$ is exponentially small in N

Entanglement is ubiquitous

S. J. Szarek: Volume of separable states is super-doubly-exponentially small in the number of qubits. Phys.Rev. A72 (2005) 032304 (https://arxiv.org/abs/quant-ph/9804024v1)

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Summary

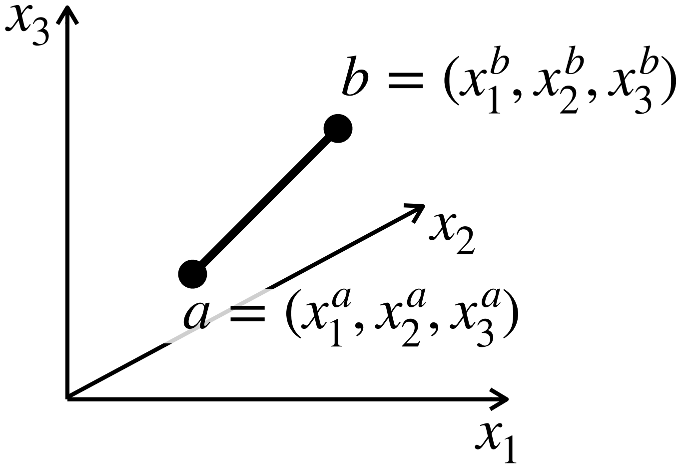
R. Oloff: *Geometrie der Raumzeit*. Springer 2018.

J. W. Robbin, D. A. Salamon: *Introduction to Differential Geometry*. Springer 2022.

R. J. Adler: *General Relativity and Cosmology*. Springer 2022.

Line Element in $\mathbb{R}^3$

(cartesian coordinates)



Distance of two points in $\mathbb{R}^3$:

$$d(a,b) = \sqrt{(x_1^b - x_1^a)^2 + (x_2^b - x_2^a)^2 + (x_3^b - x_3^a)^2}$$

$$\Rightarrow d(a,b)^2 = (x_1^b - x_1^a)^2 + (x_2^b - x_2^a)^2 + (x_3^b - x_3^a)^2$$

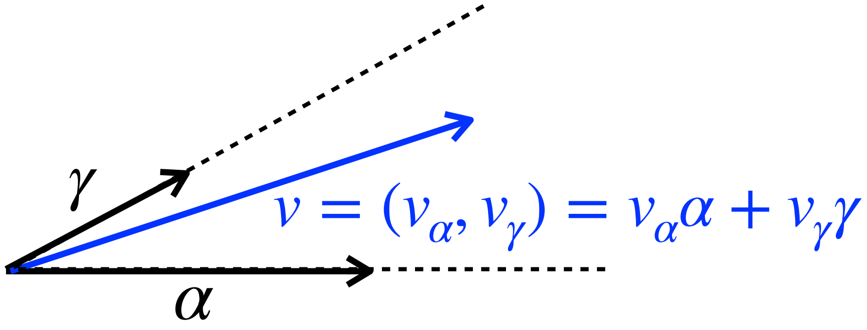
$$= \Delta x_1^2 + \Delta x_2^2 + \Delta x_3^2$$

The so-called *line element ds* is measuring infinitesimal distances:

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2$$

Line Element in $\mathbb{R}^2$

(arbitrary coordinates)



$$v = (v_\alpha, v_\gamma) = v_\alpha \alpha + v_\gamma \gamma$$

In $\mathbb{R}^n$:

$$ds^2 = \sum_{i,j=1}^{n} g_{ij}\, dx_i dx_j$$

$$\begin{aligned} \| v \|^2 = \langle v, v \rangle &= \left\langle v_\alpha \alpha + v_\gamma \gamma, v_\alpha \alpha + v_\gamma \gamma \right\rangle \\ &= v_\alpha^2 \langle \alpha, \alpha \rangle + v_\alpha v_\gamma \langle \alpha, \gamma \rangle + v_\gamma v_\alpha \langle \gamma, \alpha \rangle + v_\gamma^2 \langle \gamma, \gamma \rangle \\ &= v_\alpha^2 \langle \alpha, \alpha \rangle + 2 v_\alpha v_\gamma \langle \alpha, \gamma \rangle + v_\gamma^2 \langle \gamma, \gamma \rangle \\ &\overset{\text{def}}{=} g_{11} v_\alpha^2 + g_{12} v_\alpha v_\gamma + g_{22} v_\gamma^2 \end{aligned}$$

$\langle \alpha, \gamma \rangle = \langle \gamma, \alpha \rangle$

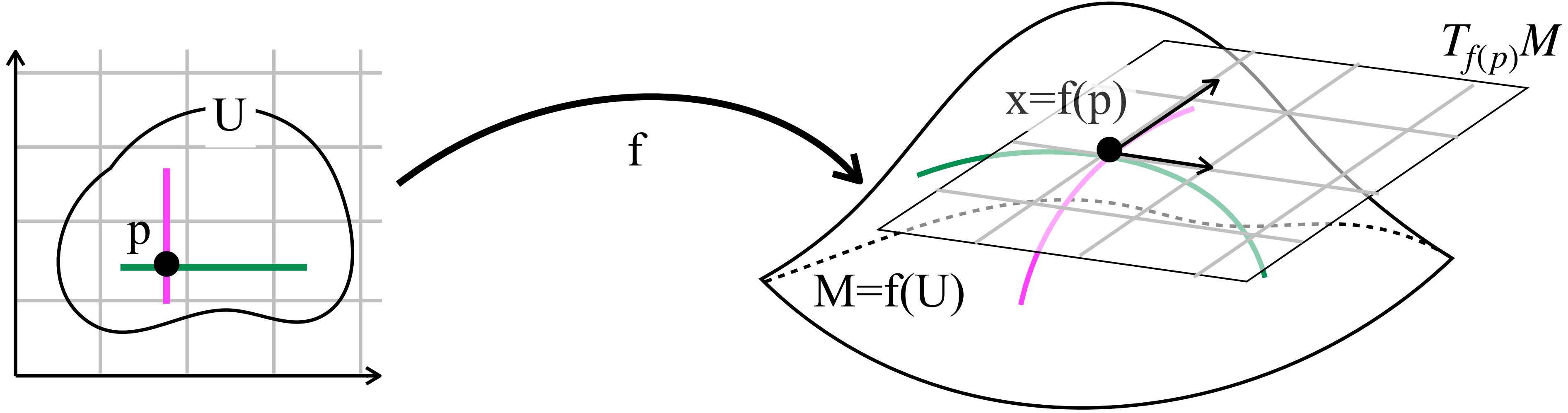
(note: $\langle \alpha, \gamma \rangle = \langle \gamma, \alpha \rangle \Rightarrow \langle \alpha, \gamma \rangle + \langle \gamma, \alpha \rangle = 2 \langle \alpha, \gamma \rangle = g_{12}$)

The *line element ds* gives the length of infinitesimal vectors (dx_1, dx_2):

$$ds^2 = g_{11} dx_1^2 + g_{12} dx_1 dx_2 + g_{22} dx_2^2$$

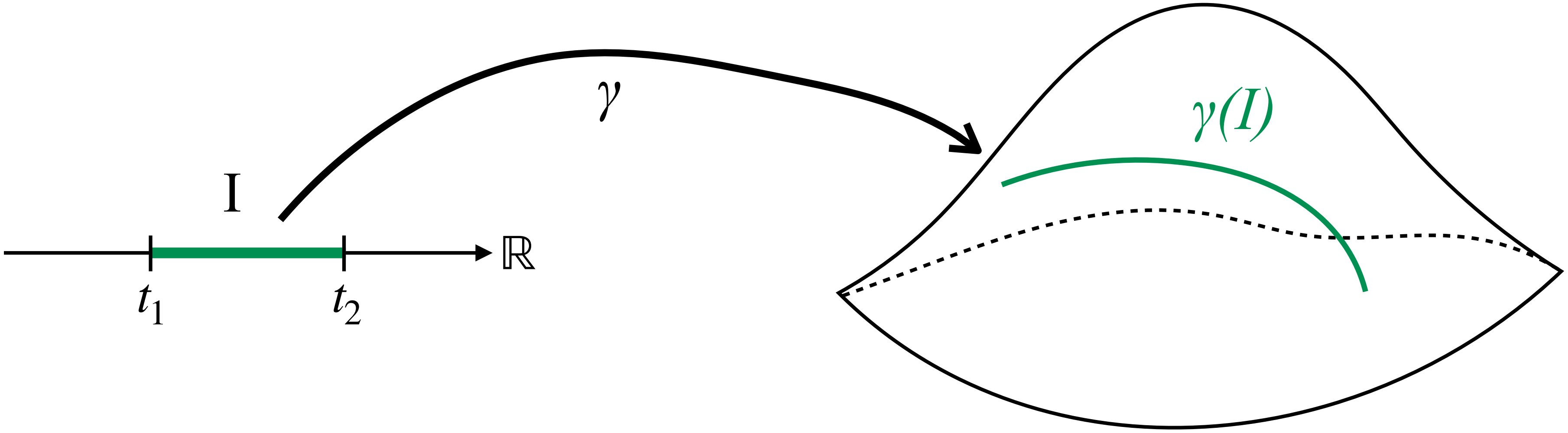
The matrix (g_{ij}) ist called *first fundamental form*

Line Element in Curved Spaces



- M=f(U) is a curved surface (*manifold*)
- Tangent vectors in x=f(p) build the *tangent space* T_xM of M in x
- First fundamental form depends on x: $(g_{ij}(x))$ (also: g_x)
- g_x defines a scalar product on T_xM : $\langle v,w\rangle_x \overset{\text{def}}{=} v^T(g_{ij}(x))w \overset{\text{def}}{=} g_x(v,w)$
  - Reminder: a scalar product induces a metric on a vector space
  - Thus, $g_x = (g_{ij}(x))$ is also called *Riemannian Metric* on M
- (M, g) is called *Riemannian Manifold*

Measuring Lengths



$$s(\gamma) = \int_{t_1}^{t_2} \sqrt{g_{\gamma(t)}\left(\gamma'(t), \gamma'(t)\right)}\, dt$$

$s(\gamma)$ is called *arc length* of γ or simply *length* of the curve γ

Reminder: Directional Derivative

Let $U \supseteq \mathbb{R}^n$, $f : U \to \mathbb{R}$, $F : U \to \mathbb{R}^m$ and $v \in \mathbb{R}^n$

$D_v f = \lim\limits_{h \to 0} \dfrac{f(x+hv) - f(x)}{h}$ is *directional derivative* of f in direction of v

- Writing: $D_v f = \partial_v f = \nabla_v f = \dfrac{\partial f}{\partial v}$

$$\boxed{D_v f = \operatorname{grad} f \cdot v \quad \text{and} \quad D_{e_i} f = \frac{\partial f}{\partial x_i}}$$

$D_v F$ ($= \partial_v F = \nabla_v F$) is build component-wise, $D_v F = DF \cdot v \ \in \mathbb{R}^m$

with Jacobi-matrix $DF \overset{\text{def}}{=} \left(\dfrac{\partial f_i}{\partial x_j} \right)_{1 \le i \le m,\, 1 \le j \le n}$

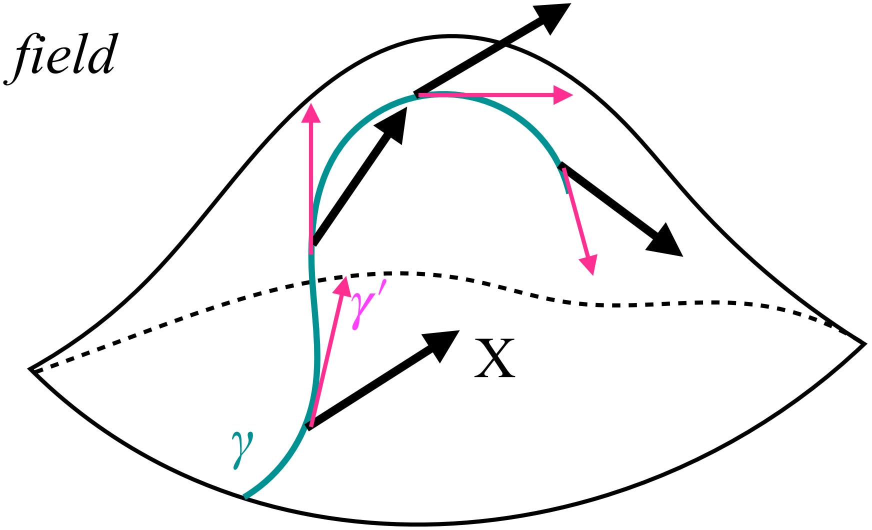
Parallel Transport

$X : M \to \bigcup_{p\in M} T_pM \overset{\text{def}}{=} TM, X(p) \in T_pM$ is *vector field*

Let $\gamma : I = [t_0, t_1] \to M$ be a curve on M.

X is *parallel along* γ :⇔ $\nabla_{\gamma'(t))}X\left(\gamma(t)\right) = 0$

(≈ X doesn't change along the curve - ∇ is directional derivative)



$$\boxed{\forall v_0 \in T_{\gamma(t_0)}M \ \exists! V : M \to TM \ : \ V(\gamma(t_0)) = v_0 \wedge \text{V parallel along } \gamma}$$

$P_{\gamma(t_0),\gamma(t_1)} : T_{\gamma(t_0)}M \to T_{\gamma(t_1)}M \ : \ v_0 \mapsto V(\gamma(t_1))$ is called *parallel transport* of v_0

Christoffel-Symbols

Let M be an n-dimensional manifold, i.e. dim T_pM = n , $\forall\, p \in M$

Let $X_1, \ldots, X_n$ be vector fields that are a basis for T_pM, $\forall\, p \in M$

Then $\nabla_{X_i} X_j = \sum_{k=1}^{n} \Gamma_{ij}^{k} X_k$, with the *Christoffel-Symbols* $\Gamma_{ij}^{k} \in \mathbb{R}$

It is: $\Gamma_{ij}^{k} = \frac{1}{2} \sum_{r} g^{kr} \left(\frac{\partial g_{jr}}{\partial x_i} + \frac{\partial g_{ir}}{\partial x_j} - \frac{\partial g_{ij}}{\partial x_r} \right)$ with $\left(g^{kr}\right) \overset{\text{def}}{=} \left(g_{kr}\right)^{-1}$

(d.h. the Christoffel-Symbols are determined by the partial derivatives of the metric)

Directional Derivatives and Christoffel-Symbole

Let $X_1, \ldots, X_n$ be vector fields that are a basis for T_pM

Let $X = \sum x_i X_i$ and $Y = \sum y_j X_j$ be arbitrary vector fields

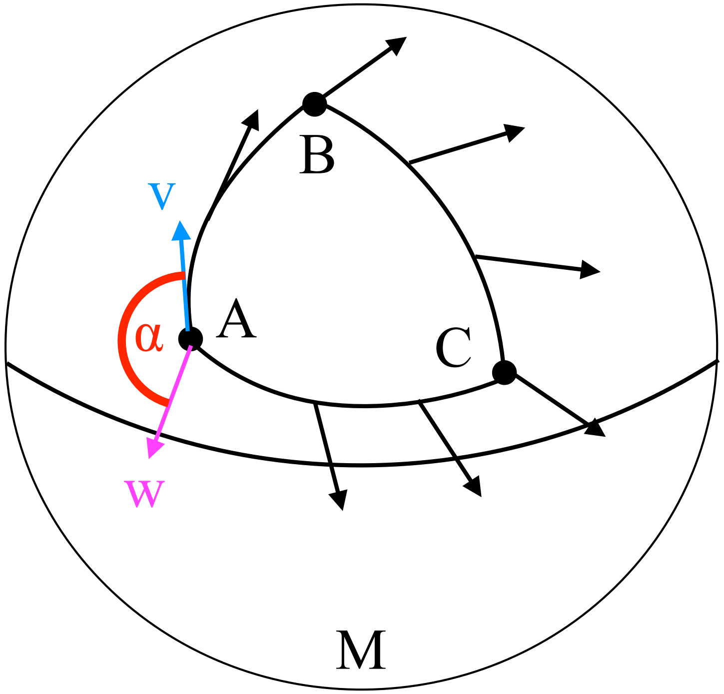
Then: $\nabla_X Y = \sum z_k X_k$

It is: $z_k = \sum_{i,j} \Gamma^k_{ij} x_i y_j + \sum_i x_i \nabla_{X_i} y_k$

(d.h. the directional derivatives are determined by the Christoffel-Symbols)

(…and, thus, by the partial derivatives of the metric)

Holonomy



The tangent vector v at A of curve AB is parallel transported along AB

The resulting vector at B is parallel transported along curve BC

The resulting vector at C is parallel transported along curve CA

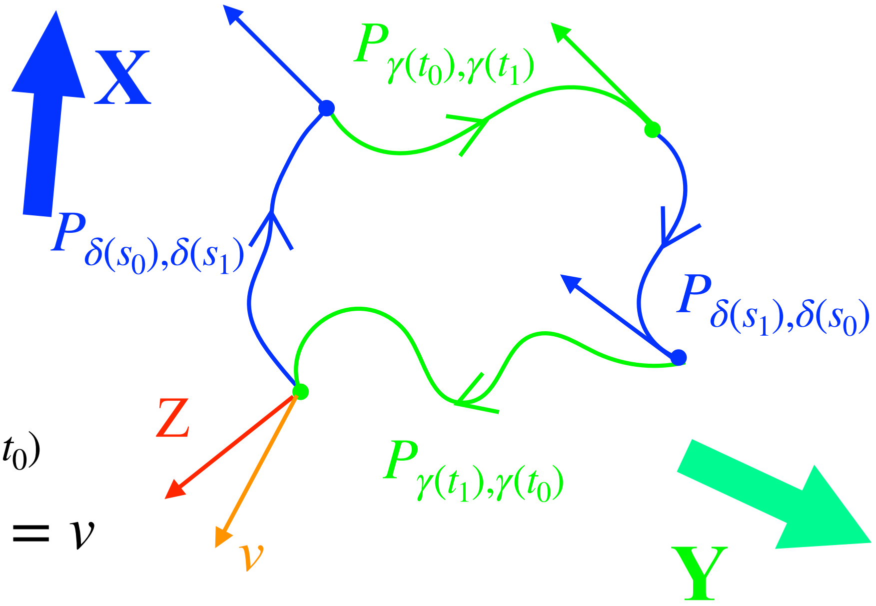
The resulting vector w at A is in general not the original vector v!

The angle α between v and w is a measure for the curvature of M

This phenomenon is referred to as *holonomy*

Riemannian Curvature

- Let X, Y, Z be vector fields
- δ, γ the curves defined by X, Y
- Z is parallel transported along
  - … X with $P_{\delta(s_0),\delta(s_1)}$ and Y with $P_{\gamma(t_0),\gamma(t_1)}$
  - … −X with $P_{\delta(s_1),\delta(s_0)}$ and −Y with $P_{\gamma(t_1),\gamma(t_0)}$



$$\Rightarrow P_{\gamma(t_1),\gamma(t_0)} \circ P_{\delta(s_1),\delta(s_0)} \circ P_{\gamma(t_0),\gamma(t_1)} \circ P_{\delta(s_0),\delta(s_1)}(Z) = v$$

- Holonomy ⇒ $v \neq Z(\delta(0))$ ⇒ curvature!
- Via $s_1 \to s_0$ and $t_1 \to t_0$ the curves become infinitesimal and an indicator of the curvature at q = $\delta(s_0)$ results

$$\Rightarrow \lim_{t_1\to t_0} \lim_{s_1\to s_0} P_{\gamma(t_1),\gamma(t_0)} \circ P_{\delta(s_1),\delta(s_0)} \circ P_{\gamma(t_0),\gamma(t_1)} \circ P_{\delta(s_0),\delta(s_1)}(Z) = \left(\nabla_X\nabla_Y - \nabla_Y\nabla_X - \nabla_{[X,Y]}\right) Z \overset{\text{def}}{=} R(X,Y)Z$$

No assumptions are being made about a space "embracing" the manifold M: *inner geometry*! Otherwise: *outer geometry*!

Ricci Curvature

R(X,Y)Z defines for $p \in M$ a multi-linear map

$$\mathrm{Rm}_p : T_pM \times T_pM \times T_pM \to T_pM$$

as follows: for $Y, Z \in T_pM$ fixed, define $\Phi_p^{Y,Z} : T_pM \to T_pM$ as
$\Phi_p^{Y,Z}(X) := \mathrm{Rm}_p(X, Y, Z) = R(X, Y)Z$

Then, $\mathrm{Ric}_p : T_pM \times T_pM \to \mathbb{R}, (Y, Z) \mapsto \mathrm{Tr}\,\Phi_p^{Y,Z}$ is called *Ricci-Map*

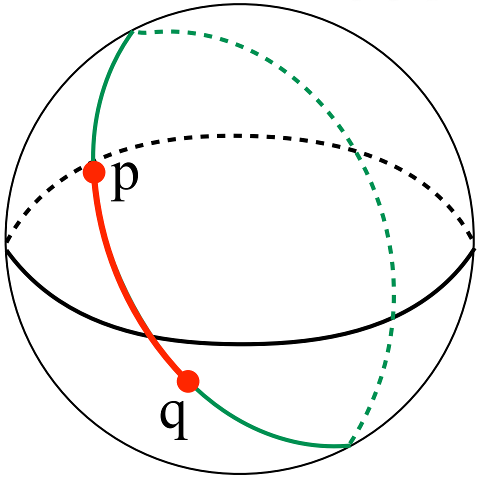
With $X \in T_pM, \parallel X \parallel = 1$, $\mathrm{Ric}_p(X, X)$ is the *Ricci-Curvature* in direction of X

Define $R_{ij} := \sum_{a=1}^{n} \frac{\partial \Gamma_{ij}^{a}}{\partial x_a} - \sum_{a=1}^{n} \frac{\partial \Gamma_{ai}^{a}}{\partial x_j} + \sum_{a=1}^{n} \sum_{b=1}^{n} \left(\Gamma_{ab}^{a} \Gamma_{ij}^{b} - \Gamma_{ib}^{a} \Gamma_{aj}^{b} \right)$

(d.h. the R_{ij} are defined by means of the Christoffel-Symbole and their derivatives, i.e. by the metric)

Then $R := \sum_{i,j} g_{ij} R_{ij}$ is called *Ricci-scalar* or *scalar curvature*

Geodesics



A curve $\gamma : I \to M$ is called *geodesic* $:\Leftrightarrow \nabla_{\gamma'}\gamma' = 0$

- The tangent vector γ' does not change along the curve
- The curve has no curvature within M

Locally, a geodesic is always the shortest connection between two points

(A geodesic on a sphere is always a segment of a great-circle:
the red geodesic is the shortest connection between p and q, but the green geodesic connects p and q too [but is not the shortest connection])

A curve $\gamma = (\gamma_1, \ldots, \gamma_n)$ is a geodesic ⇔ For 1≤i≤n:

$$\frac{d^2\gamma_i}{dt^2} + \sum_{j,k} \Gamma^i_{jk} \frac{d\gamma_j}{dt}\frac{d\gamma_k}{dt} = 0$$

(Reminder: the Γ^i_{jk} are determined by the metric and its partial derivatives!)

Einstein’s Field Equation

$$\boxed{\overbrace{R_{\mu\nu}}^{\textit{Curvature}} \overbrace{- \frac{1}{2}Rg_{\mu\nu}}^{\textit{Geodesics}} = \overbrace{\frac{8\pi G}{c^4}T_{\mu\nu}}^{\textit{Matter}}}$$

Ricci Tensor

Metric

Stress-Energy Tensor

Scalar Curvature

Matter (Stress-Energy Tensor)
results in cuvature (Ricci Tensor) of space-time,
such that particles move on geodetics (metric)

(a system of 16 partial differential equations of 2nd order)

Schwarzschild-Metric

Let M be a mass, that neither rotates nor is it charged

Outside M in its nearby environment, there are no other masses

Then: $T_{\mu\nu} = 0$ (vacuum field equation)

⇒ solution (in spherical coordinates) is the so-called *Schwarzschild-Solution*:

$$\boxed{\mathrm{d}s^2 = -c^2\left(1-\frac{2GM}{c^2 r}\right)\mathrm{d}t^2 + \left(1-\frac{2GM}{c^2 r}\right)^{-1}\mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\phi^2}$$

Solution has two singularities at $r = 0$ and at $\boxed{r_S := \frac{2GM}{c^2}}$ (*Schwarzschild Radius*)

- In proper coordinates, r_S is no longer a singularity
- Singularity at $r = 0$ is a proper singularity (i.e. independent of any chosen coordinate system)

(Anti-) de Sitter

Field equation with cosmological constant: $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$

- Λ has been introduced by Einstein in order to get a static universe
- <u>But</u>: in the meantime we know that the universe is not static

New observations can be explained with Λ

- $\Lambda > 0 \Rightarrow$ anti-gravitation ("dark energy") $\Rightarrow$ expansion of the universe
- $\Lambda < 0 \Rightarrow$ contraction of the universe

Vacuum solutions with cosmological constant:

- *de Sitter* space ($\Lambda > 0$) $\Rightarrow$ constant positive curvature ($\triangleq$ sphere)
  - …matched by observations
- *Anti de Sitter* space ($\Lambda < 0$) $\Rightarrow$ constant negative curvature ($\triangleq$ saddle surface)
  - space does <u>not</u> expand
  - …does <u>not</u> match observations

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ER = EPR

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M. Camenzind: *Faszination kompakte Objekte*. Springer 2021.

W. Schmitz: *Understanding Relativity*. Springer 2022.

D. Grumiller, M. M. Sheikh-Jabbari: *Black Hole Physics*. Springer 2022.

Escape Velocity

…is the velocity v_F , which a mass m at a distance r from a mass M must have to escape from M

I.e. the kinetic energy of m must be equal to the binding energy within the gravitational field of M:

$$\frac{1}{2}mv_F^2 = \frac{GMm}{r} \quad \Rightarrow \quad v_F = \sqrt{\frac{2GM}{r}}$$

Earth: $v_F = 11{,}2$ km/s

Sun: $v_F = 617{,}4$ km/s

Event Horizon

With $r = r_S = \frac{2GM}{c^2}$ the escape velocity becomes $v_F = \sqrt{\frac{2GM}{r_S}} = c$!

⇒ From this area even light cannot escape !

⇒ This area appears to be completely black: *black hole*

Out of the area within the Schwarzschild radius, no information at all can reach us, i.e. this area is for external observers eventless.

Thus, the sphere with radius r_S is called *event horizon*

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D. Grumiller, M. M. Sheikh-Jabbari: *Black Hole Physics*. Springer 2022.

G. Musser: *Emergence in Condensed Matter and Quantum Gravity*. Springer 2022.

Cosmic Censorship

Penrose-Hawking Theorems prove:

Singularities are consequences of gravitational collapse,
all collapsed matter will be concentrated to a single point.

Do singularities exist, which are not surrounded by an event horizon, i.e. that are directly observable (so-called *naked* singularities)?

Conjecture (up to now without proof):

Naked black holes do not exist.

Surface Gravity

Gravitational acceleration g at the surface r of a mass M

results from $mg = \frac{GmM}{r^2}$ d.h. $g = \frac{GM}{r^2}$

With $r = r_S = \frac{2GM}{c^2}$ the gravitational acceleration κ_H

at the event horizon is: $\kappa_H = \frac{GM}{(2GM/c^2)^2} = \frac{c^4}{4GM}$

κ_H is called *surface gravity* at the event horizon

The surface gravity κ_H at the event horizon is constant.

Horizon Theorem

Given two black holes of masses M_1, M_2, which merge

The mass of the merged black hole is then $M = M_1 + M_2$

The Schwarzschild radius of the merged black hole is:

$$r_S = \frac{2GM}{c^2} = \frac{2G(M_1 + M_2)}{c^2} = \frac{2GM_1}{c^2} + \frac{2GM_2}{c^2} = r_{S,1} + r_{S,2}$$

The surface area of the horizon of the merged black hole is:

$$A_H = 4\pi r_S^2 = 4\pi(r_{S,1} + r_{S,2})^2 = 4\pi r_{S,1}^2 + 4\pi r_{S,2}^2 + 8\pi r_{S,1} r_{S,2}$$
$$= A_{H,1} + A_{H,2} + 8\pi r_{S,1} r_{S,2} > A_{H,1} + A_{H,2}$$

The horizon area A_H of black holes cannot shrink.

(experimental confirmation via measurements of gravitational waves in 2021)

Decay of Black Holes?

Assumption: A black hole of mass $M = M_1 + M_2$ decays

into two black holes of masses M_1, M_2

As shown before: $A_H = A_{H,1} + A_{H,2} + 8\pi r_{S,1} r_{S,2} > A_{H,1} + A_{H,2}$

$\Rightarrow$ By decay, the horizon area would shrink: contradiction!

A black hole cannot decay (into black holes of smaller masses).

No-Hair-Theorem

All properties of a black hole are completely determined
by its mass M, its angular momentum J and its electric charge C.

I.e. no other physical quantities (magnetic field, number of particles, their spin,…) influence the properties of a black hole

These additional quantities ("hair") are irrelevant for black holes

Black holes have no hair

But the particles that collapsed into the black hole had these quantities!

<u>Problem</u>: What happened to this corresponding information? (see later)

Vacuum Fluctuation

Reminder: Heisenberg Uncertainty Relation $\Delta_v(A) \cdot \Delta_v(B) \geq \frac{1}{2} \left| \left\langle [A, B] \right\rangle_v \right|$

This gives: $\Delta E \, \Delta t \geq \frac{\hbar}{2}$ mit $\hbar := \frac{h}{2\pi}$

$E = mc^2 \Rightarrow$ for a small period of time Δt a mass Δm can come into existence

Conservation laws of physics: this matter consists of particle/antiparticle pairs

≈ Conception: in the vacuum these pairs are created permanently,

but they are destroyed in a very short time ("annihilation")

"The vacuum is fluctuating"

Evaporation of Black Holes

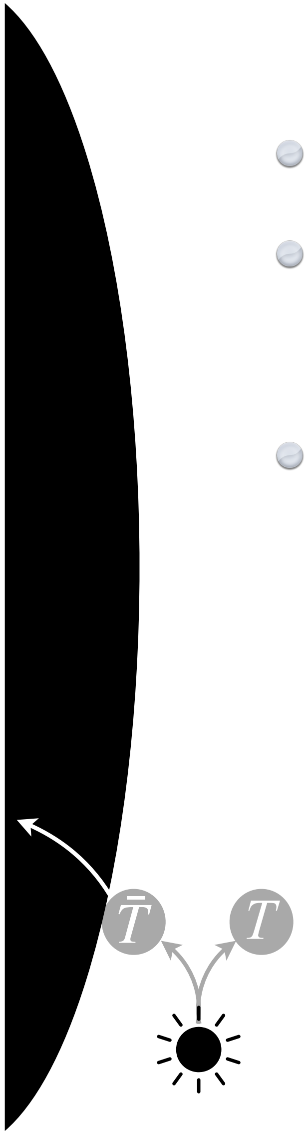
- Close to the event horizon, $T, \bar{T}$ are created because of vacuum fluctuation
- Assumption: $\bar{T}$ (or T) falls into the black hole, T (or $\bar{T}$) escapes

  ⇒ No annihilation possible: Violation of the conservation of energy!
- Analysis: The energy of $\bar{T}$ is negative (⇒ conservation of energy)!

  ⇒ $\bar{T}$ has negative mass ($E = mc^2$)

  ⇒ The black hole is loosing mass

"*Evaporation of the black hole*"



$$\boxed{t_{\text{evaporation}} = 2.1 \times 10^{67} \left(\frac{M}{M_\odot}\right)^3 \ [\text{years}]}$$

Stellar black holes ($M \geq M_\odot$) ⇒ $t_{\text{evaporation}} > 10^{67}$ [years] $\gg 1.4 \times 10^{10}$ [years] (age of the universe)

⇒ Evaporation is irrelevant at present! (but: primordial black holes!)

Macro-/Microstates

A system consists of N particles. We distinguish:

- *Microstate*: state of an individual particle of the system
- *Macrostate*: overall state of the whole system

The macrostate result from the microstates

Example: The macrostate of a gas (temperature, pressure,…) results from the microstates (location, momentum) of the gas particles.

Phase space: set of all possible microstates

Let p_i be the probability that the i-th particle is in a certain microstate

Configuration: particular probability distribution of the microstates

(each point in the phase space has an associated probability)

Entropy

Information that is needed to determine the configuration of a certain macrostate is called *entropy S*

$$S := -\, k_B \sum_{i=1}^{N} p_i \cdot \ln p_i$$

The term has its origin in thermodynamics

There it is proven: $\boxed{dS = \frac{dQ}{T}}$ (Q: amount of heat, T: temperature)

Second law of thermodynamics: $\boxed{dS \geq 0}$

Quantum Mechanical Interpretation of Entropy

The microstate of particle i is the point $|i\rangle$ in Hilbert space

- The phase space is substituted by the Hilbert space

The corresponding macrostate is the density matrix $\rho = \sum p_i |i\rangle\langle i|$

von Neumann entropy of ρ is $S := -\operatorname{Tr}(\rho \ln \rho)$

Quantum physical entropy corresponds up to the factor k_B
to the information theoretical entropy

Summary: Entropy

Entropy measures the information needed

to completely describe the macrostate of a system

based on the distribution of its microstates

- Entropy can never decrease
- Things with entropy have a temperature

Bekenstein-Hawking Formula

A body with mass m and entropy S falls into a black hole

⇒ Entropy outside the event horizon is reduced

- Because: the information content of the body is no longer available outside!

⇒ The mass and, thus, the horizon area of the black hole increases

$$r_S = \frac{2GM}{c^2}$$

$$A_H = 4\pi r_S^2$$

Because entropy cannot decrease, the otherwise lost entropy must correspond to the increase of the horizon area

⇒ The horizon area is a measure for the entropy of the black hole

The entropy of a black hole is $$\boxed{S = \frac{k_B}{4\mathfrak{L}_P^2} A_H}$$

with the *Planck Length* $\mathfrak{L}_P = \sqrt{G\hbar / c^3}$ (≈ $1{,}616 \cdot 10^{-35}$ m)

(smallest length in which space can be subdivided; everything smaller that $\mathfrak{L}_P$ collapses to a black hole)

Hawking Radiation

Remember: Things with entropy have a temperature

Temperature of a black hole is $T = \frac{\hbar c^3}{8\pi G M k_\mathrm{B}} = \frac{\hbar}{2\pi c k_B}\kappa_H$

- Note: $T \propto \frac{1}{M}$

But a body with a temperature emits thermal radiation
⇒ a black hole radiates: contradiction(!)
because nothing can escape a black hole!

<u>Solution</u>: this radiation corresponds to the particles, that are created by the evaporation of the black hole

⇒ By radiating, the black hole loses mass, thus, it gets hotter until it explodes!

Holographic Principle

(t’Hooft, Susskind)

Information content of a black hole
proportional to the number of potential microstates within the black hole

Thus, information content should be proportional to the volume of the black hole

- …because the particles are scattered across the volume of the black hole

But the information content is proportional to the horizon area $S = \frac{k_B}{4\mathfrak{L}_P^2} A_H$

- Via entanglement the number of degrees of freedom in the volume becomes proportional to its enclosing surface(*)
- The degrees of freedom of the microstates correspond 1−1 to the degrees of freedom at the horizon : *Holographic Principle*

The horizon is like a hologram of the inner of the black hole

- All information about the inner is encoded on the horizon

L. Susskind: The world as a hologram. arXiv:hep-th/9409089v2 (1994)

(*) W. Schmitz: Understanding Relativity. Springer 2022 — Ch. 15.1

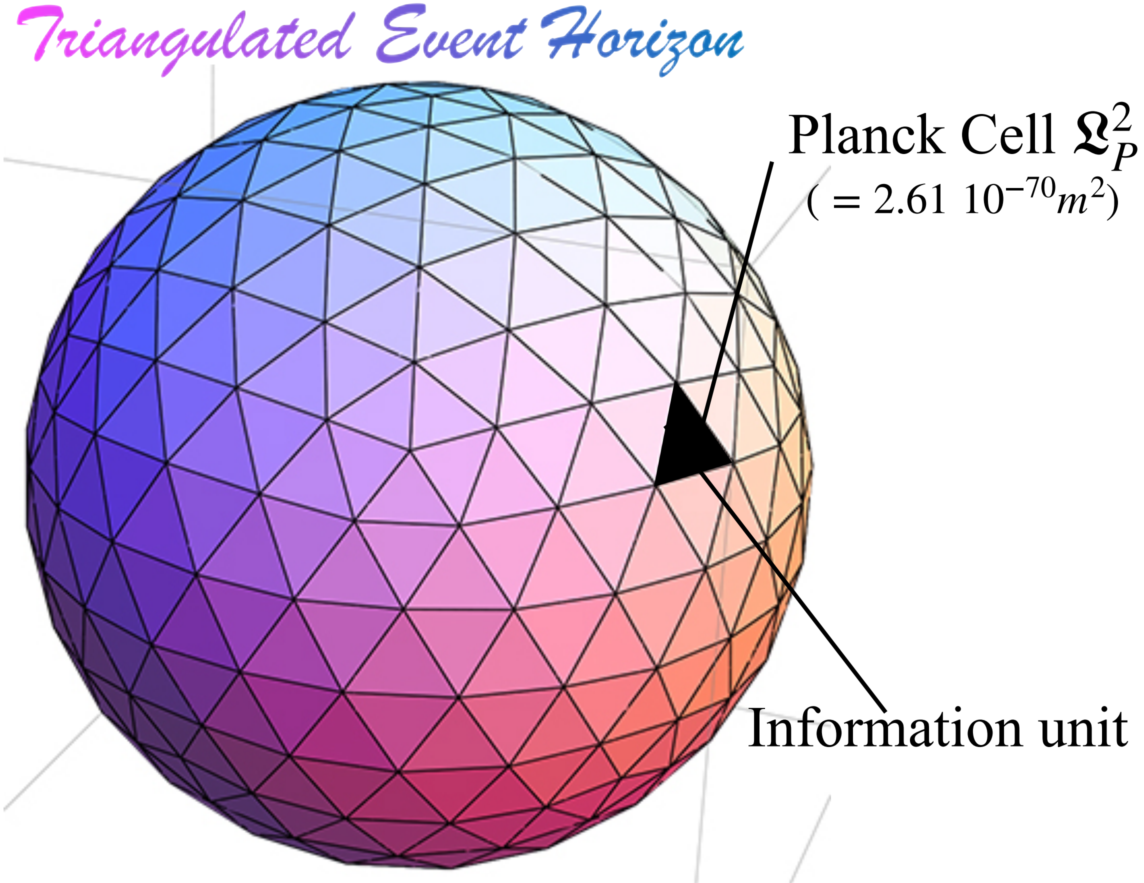
Horizon Area & Information

$$S = \frac{k_B}{4}\frac{A_H}{\mathfrak{L}_P^2}$$

⇒ Entropy is the number of Planck Cells $A_H/\mathfrak{L}_P^2$ on the horizon (up to a factor)

Planck Cell $\mathfrak{L}_P^2$ is the smallest area, which can carry information:
Planck Cell ≙ 1 Qubit

"*Information equates surface*"



(Quelle: https://mathoverflow.net/q/144405)

The same can be shown for the horizon of the whole universe

"It from Qubit"

Spacetime Quantum Information

Information & Gravitation

$$S = \frac{k_B}{4\mathfrak{L}_P^2} A_H$$

- The event horizon is just thus big to fit the information content of all matter that has fallen into the black hole on its horizon

- This information content determines the horizon area and, thus, the Schwarzschild radius $r_s = \sqrt{\frac{A_H}{4\pi}} = \frac{\mathfrak{L}_P}{\sqrt{\pi k_B}}\sqrt{S}$

- This information content determines also the surface gravity $\kappa_H = \frac{GM}{r_S^2}$

Interpretation

The entropy and, thus, the information content of a black hole determines the curvature of space-time "close" to the black hole

Gravity does not exist at microscopic scales
(Planck Length)

Gravity is a macroscopic effect of entropy and information

Space-time consists of smallest structures (Planck Cells),
each of which carry 1 Qubit of information

The qubits of the Planck Cells are entangled

(W. Schmitz: Understanding Relativity. Springer 2022 — Ch. 11.5)

Spacetime via Emergence

Spacetime is a phenomenon of emergence, it is no longer fundamental

Spacetime is a fabric, it emerges from entangling qubits that represent Planck cells(*)

(but nobody knows yet what these qubits are made of)

"It from Qubit"

(*) B. Swingle: Spacetime from entanglement. Ann.Rev.Condensed Matter Phys. 9 (2018)

Information Paradox

General Relativity Theory

Before falling into a black hole, particles have more properties than just mass, angular momentum J and its electric charge — i.e. those quantities that completely determine the properties of a black hole (No-Hair Theorem)

- Like spin,…

This additional information is lost when passing the event horizon

When a black hole evaporates, this information is not recovered

Quantum Physics

The collapse is a unitary process, thus, reversible. Especially, the information lost can be recovered!

This is a contradiction!

Additional Argument: No-Hiding Theorem

When information is lost from a subsystem $\mathbb{H}_A$,
which is part of the composed system $\mathbb{H}_A \otimes \mathbb{H}_B$,
this information moves to the subsystem $\mathbb{H}_B$.

Information within a closed system is never lost
(≡ Law of conservation of quantum information)

Braunstein, S. L., & Pati, A. K. Quantum information cannot be completely hidden in correlations: Implications for the black hole information paradox. Phys. Rev. Lett. 98, 080502 (2007).

Agenda

Entanglement

Einstein’s Field Equation

Black Holes

Properties of Black Holes

ER = EPR

Summary

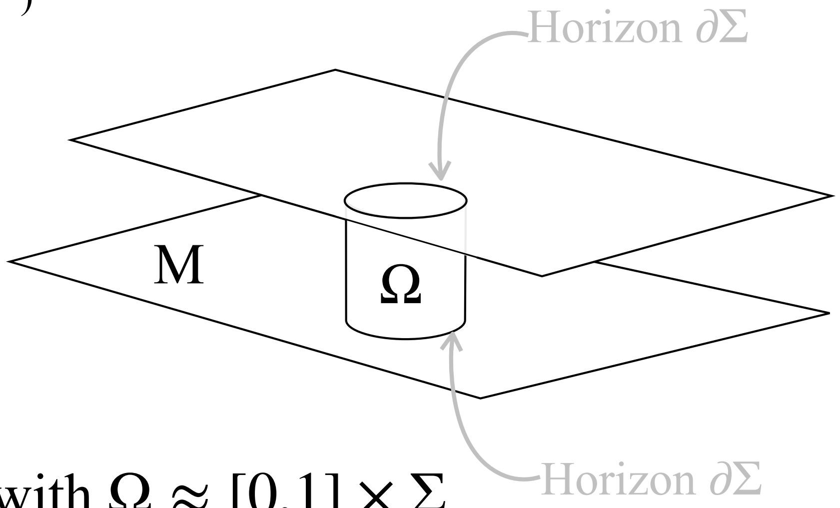
J. Maldacena, L. Susskind: Cool horizons for entangled black holes. arXiv:1306.0533v2 (2014)

Wormhole

Introduced by H. Weyl as a possible construct(*)

≡ Special solution of Einstein's Field Equation(**)

- Space-time consists of two "sheets"
- The "sheets" are connected by a "surface of a cylinder"



More precise:

Let M be the space-time and $\Omega \subseteq_{\text{compact,conn}} M$ with $\Omega \approx [0,1] \times \Sigma$

and Σ be a 3-manifold with $\partial\Sigma \approx \mathbb{S}^2$,

then Ω is called a *wormhole* in M

$\{0\} \times \partial\Sigma$ and $\{1\} \times \partial\Sigma$ are the horizons of the two connected black holes

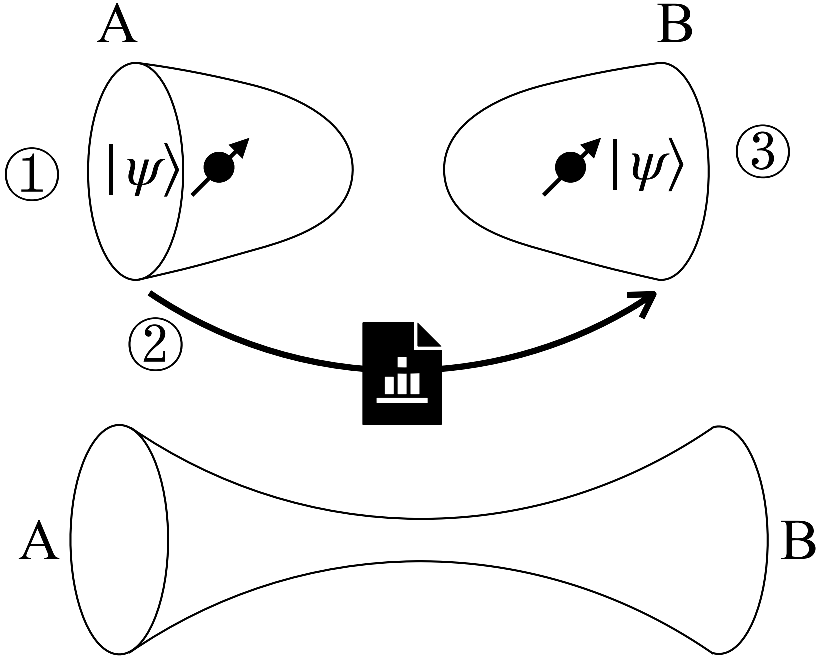
(*) H. Weyl: "Feld und Materie". Annalen der Physik. 65 (14), (1921)

(**) A. Einstein, N. Rosen: The Particle Problem in the General Theory of Relativity. Phys. Rev. 48, 73 (1935)

A First Indicator

Quantum-mechanical computations allow:

① State $|\psi\rangle$ is created in a black hole A
② "Some" measurements are performed at the horizon of A, and the results are send to B via classical communication
③ This data encodes a process, that is performed at B: as a result, $|\psi\rangle$ appears in the black hole B

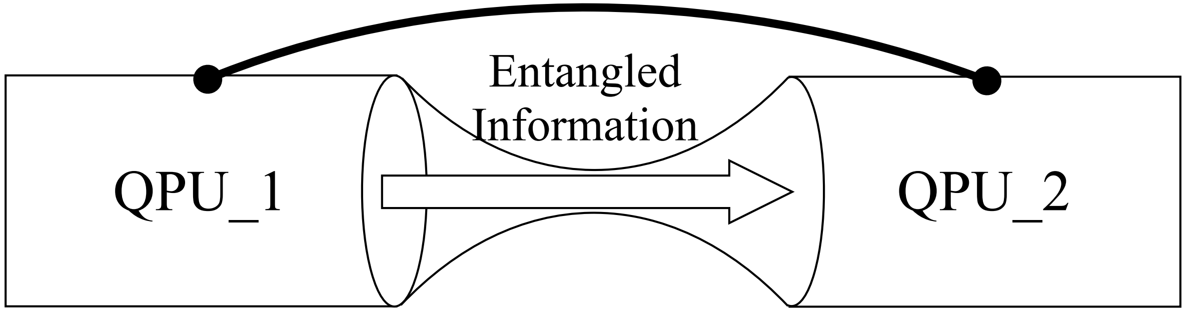


⇒ In order for $|\psi\rangle$ to get from A to B, there must be a connection between A and B: a wormhole!

Note the similarity to teleportation, which requires entanglement between A and B

Wormholes and entanglement are equivalent descriptions

Another Indicator



- QPU_1 and QPU_2 are entangled
- Information is submitted from QPU_1 to QPU_2
- The state of QPU_1 influences what will actually be submitted
- Thus, QPU_2 can learn about the state of QPU_1
- Or: at the "2-end" of the wormhole one can learn about the first black hole

Computation with gravitational theory (i.e. wormholes)
and computations with quantum protocols (i.e. entanglement)
deliver the same results

ER = EPR

Wormholes(*) [**ER**] A. Einstein, N. Rosen: The Particle Problem in the General Theory of Relativity. Phys. Rev. 48, 73 (1935)

Entanglement [**EPR**] A. Einstein, B. Podolsky, N. Rosen: Can quantummechanical description of physical reality be considered complete? Phys. Rev. 47, 777 (1935)

(*) Wormholes are also called *Einstein-Rosen-Bridges*

ER = EPR:
Wormholes and entanglement are the same phenomenon

J. Maldacena, L. Susskind: Cool horizons for entangled black holes. arXiv:1306.0533v2 (2013)
J. Maldacena et al.: Diving into traversable wormholes. Fortschr. Phys., 65(5), 1700034 (2017)

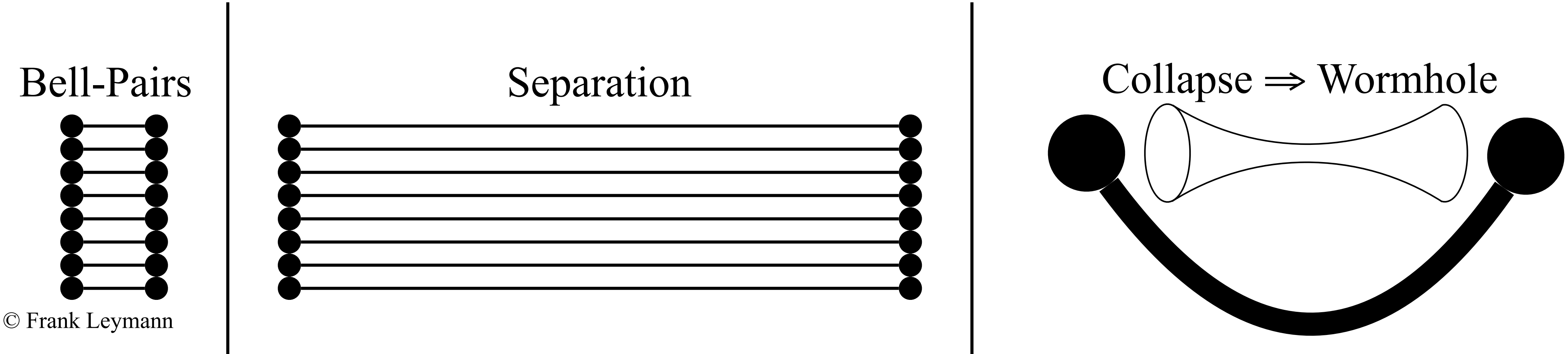
Consequence: Entanglement between particles origin from wormholes!

J.C. Baez, J. Vicary: Wormholes and Entanglement. Classical and Quantum Gravity, Vol. 31 No. 21 (2014) arXiv:1401.3416v2

Generation of Wormholes in Principle

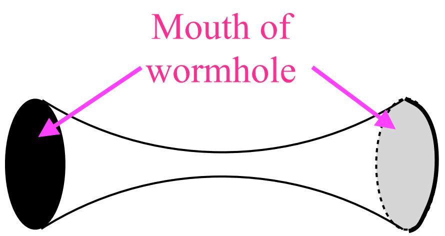
- Create a huge set of Bell-pairs
- Split the entangle particles into two sets and move them apart in large distance
- Then, collapse each of these particle clouds into a black hole

⇒ Two entangled black holes!

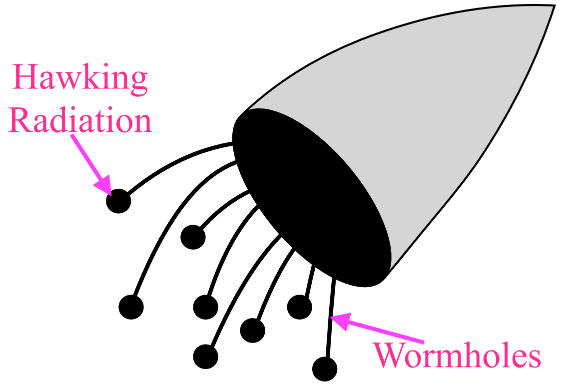


Hawking Radiation & Wormholes

- Black holes resulting from a "simple" collapse are unilateral, i.e. they are not *mouth* of a wormhole



- Particles resulting from evaporation of a black hole are entangled with the inner black hole
  - …and, thus, connected with the black hole by means of "small" wormholes

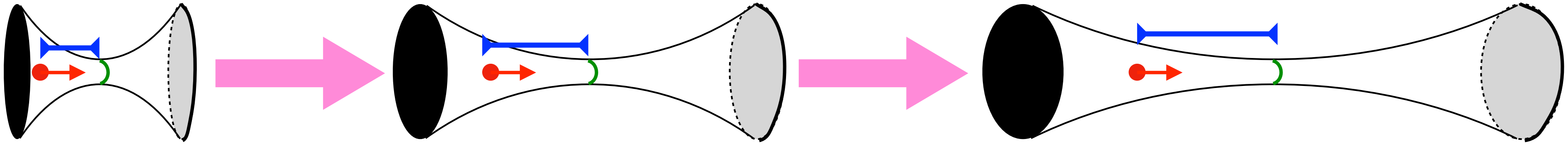


- By collapse of (a subset of) the emitted particles a "big" wormhole with two mouths results
  - …time required for this process is called Page-Time

Wormhole Traversability(*)

A wormhole cannot be traversed

- A wormhole is growing faster than matter can move through it(*)
- This can be explained via complexity theory (complexity of a circuit needed to create the quantum state of the black hole)



But one can "meet in the middle"

Microscopic wormholes could be traversable

- …and submit information

(*) L. Susskind: Three Lectures on Complexity and Black Holes. SpringerBriefs in Physics (2020).

Experiment: Traversable Wormhole(*)

Based on *holographic duality*, i.e.:

- Dynamics of quantum systems ≈ Effects of quantum gravity

  ⇒ With QPUs one can run experiments about quantum gravity(**)

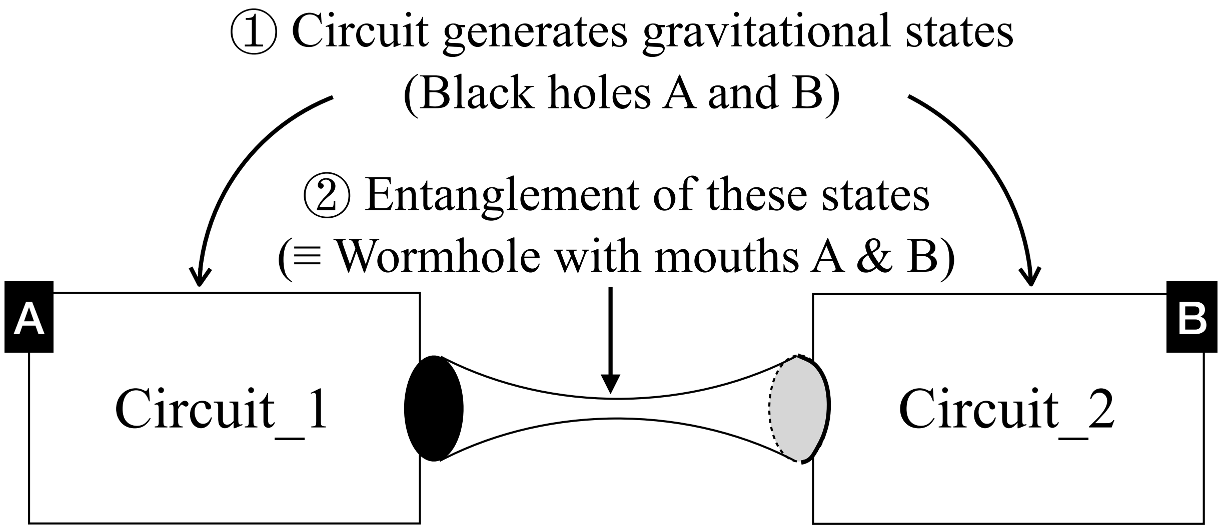
- This enables quantum circuits whose execution correspond to the dynamics of traversable wormholes(*)
  - Effects of negative energy ≡ Operations on entangled qubits

This is consistent with ER = EPR !

- This can be understood as a kind of quantum teleportation

(*) D. Jafferis et al.: Traversable wormhole dynamics on a quantum processor. Nature volume 612, pages 51–55 (2022)
[BLOG Summary 1: https://ai.googleblog.com/2022/11/making-traversable-wormhole-with.html]
[BLOG Summary 2: https://inqnet.caltech.edu/wormhole2022/]

(**) S. Nezami et al.: Quantum Gravity in the Lab: Teleportation by Size and Traversable Wormholes (2021)
Part 1 arXiv:1911.06314v2 — Part II arXiv:2102.01064v1



① Circuit generates gravitational states (Black holes A and B)

② Entanglement of these states (≡ Wormhole with mouths A & B)

③ Creation of two entangled qubits (ERP pair) at A (probe and reference)

④ SWAP the probe with a qubit in B (≙ probe in wormhole)

⑤ Circuit with chaotic evolution of the wormhole (probe gets "scrambled")

⑥ Entanglement operation between A and B (≙ negative energy ⇒ traversability)

⑦ Another circuit with chaotic evolution of the wormhole (moves probe to B)

⑧ Determine degree of entanglement between probe_B in B and reference_A in A

- Degree of entanglement has increased ⇒ information has been submitted!

Within holographic duality this means:
The probe is a particle, which has been moved from A to B

Agenda

Entanglement

Einstein’s Field Equation

Black Holes

Properties of Black Holes

ER = EPR

Summary

Summary

- Einstein’s Field Equation describes the inner geometry of space-time
- Black holes result as a solution of this equation
  - …and are confirmed by observation
- Black holes have fundamental properties
  - Horizon Theorem, No-Hair Theorem, Haking-Radiation,…
- Entropy is proportional to the area of the event horizon

  ⇒ Holographic Principle
- Wormholes are a special solution of the field equation
  - …and can be described as entanglement between black holes
- NISQ enables experiments about quantum gravity
- Deep understanding of…
  - Entanglement ⇒ ER = EPR
  - Spacetime (emergence from Planck-Cells) ⇒ *It from Qubit*

End